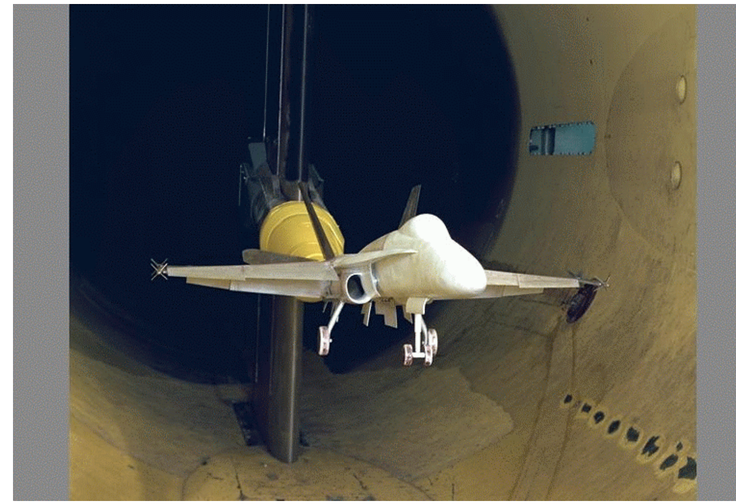
FE.1 Engineering Simulation

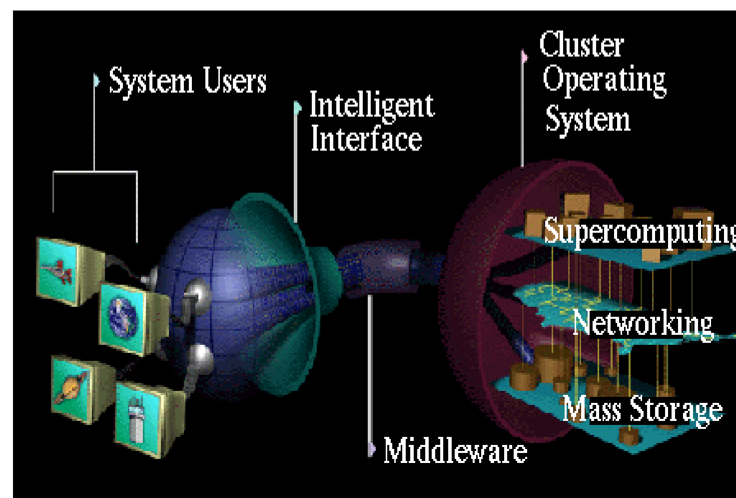
Physical Laboratory:

- ***model*** the geometry
  - similitude
  - cost
- ***measure*** the data
  - interpolation (errors)
  - ***interpretation***



Computational Laboratory:

- ***model*** the mathematics
  - conservation, BCs
- ***model*** the physics
  - complexity, cost
- ***compute*** the data
  - approximation error
  - physics model error
  - ***interpretation***



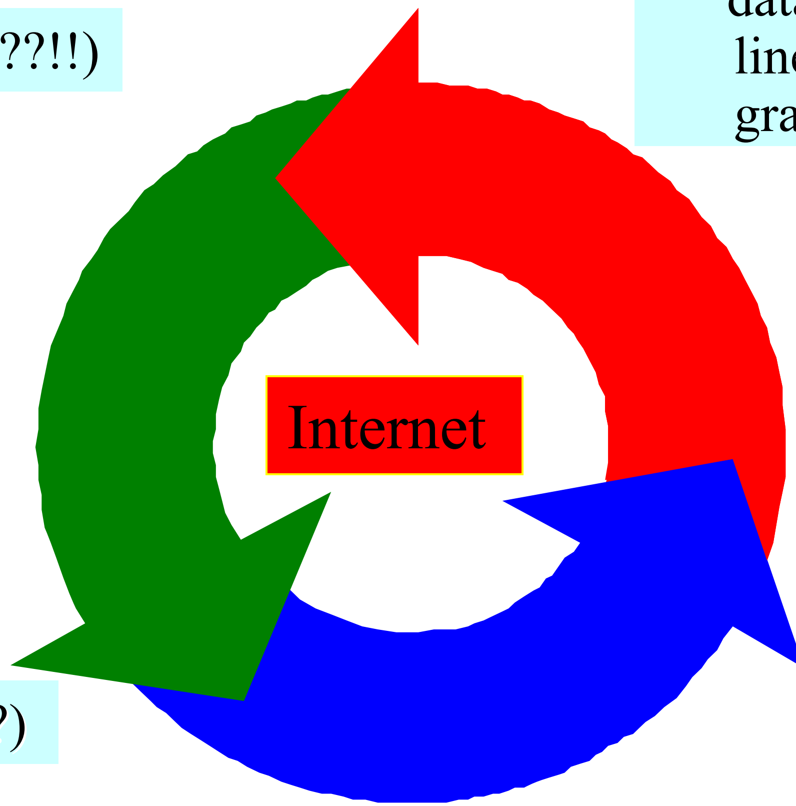
FE.2 A Problem Solving Environment

Computer Science

computer platforms
data management
linear algebra
graphics

knowledge (??!!)

Engineer
specified problem



knowledge (??)

knowledge (?)

Mathematics / Physics

conservation principles
physics closure models
PDEs + BCs + ICs, discrete methods

FE.3 Problem Statements in Engineering

Unknown $q(\mathbf{x})$ satisfies a PDE

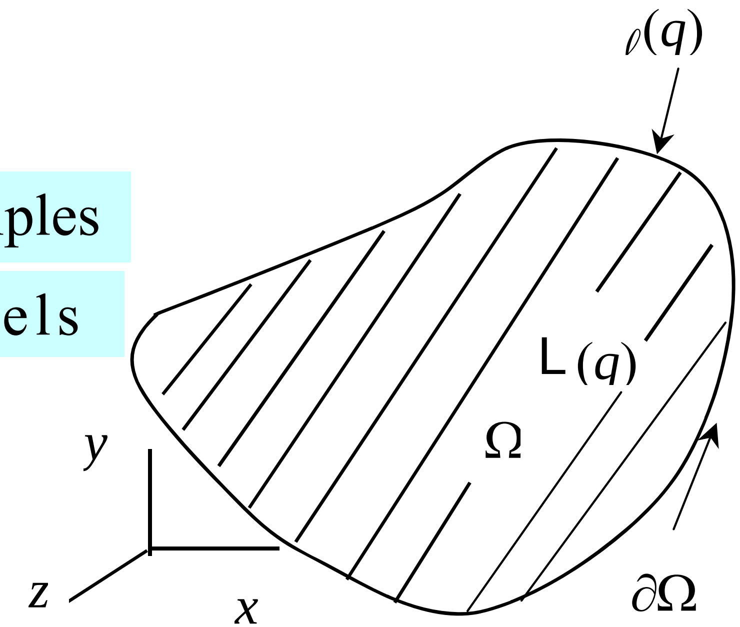
$$\mathrm{L}(q) = 0, \;\; on \;\; \Omega \subset \Re^n$$

e.g., mass, momentum, energy principles

+ physics closure models

Connection to specifics involves BCs

$$\ell(q) = 0, \;\; on \;\; \partial\Omega \subset \Re^{n-1}$$



Non-linearity, geometry preclude analytical solution

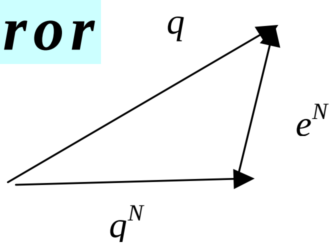
identify mathematically an approximation

$$q(\mathbf{x}) \approx q^N(\mathbf{x}) = \sum_{\alpha}^{N} \Psi_\alpha(\mathbf{x})\, Q_\alpha$$

FE.4 FE Discrete Solution Process

Exact and approximate solutions differ by *error*

$$q(\mathbf{x}) = q^N(\mathbf{x}) + e^N(\mathbf{x})$$



Galerkin weak statement *minimizes* the error!

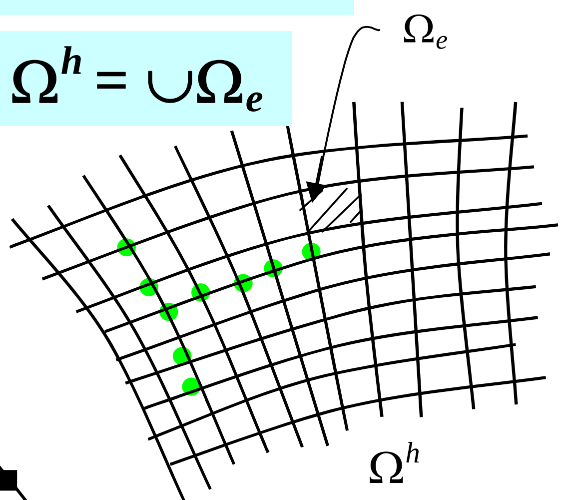
$$\mathrm{GWS}^N \equiv \int_\Omega \Psi_\beta(\mathbf{x}) \mathrm{L}(q^N) d\tau \equiv 0, \text{ for } \textit{all} \text{ trial functions } \Psi_\beta$$

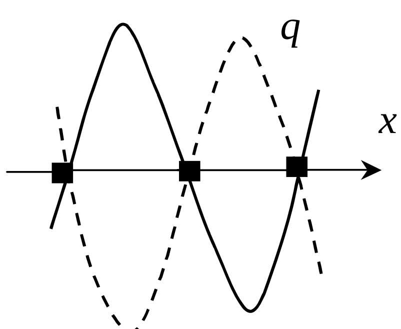
Discretize Ω into finite elements: $\Omega \Rightarrow \Omega^h = \cup \Omega_e$

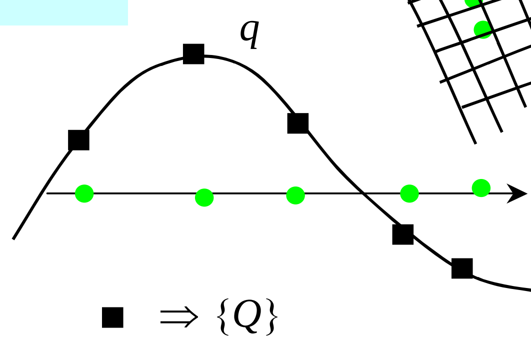
notationally: $q^N \equiv q^h = \cup_e q_e(\mathbf{x})$

$\mathrm{GWS}^N \equiv \mathrm{GWS}^h \Rightarrow \{Q\}$ at nodes

$\Rightarrow$ mesh resolution requirement







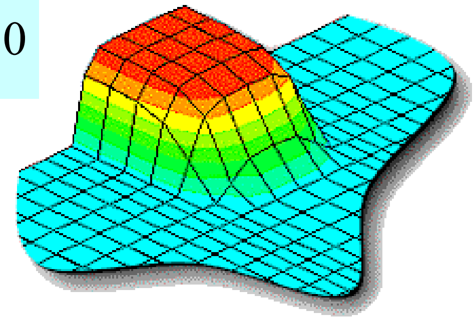
FE.5 Summary, Finite Element Analysis

For arbitrary geometries and non-linearity

problem statement: $\mathrm{L}(q) = 0 \quad on \quad \Omega \subset \Re^n \quad + \quad \text{BCs}$

approximation: $q(\mathbf{x}) \approx q^N(\mathbf{x}) \equiv \sum_{\alpha}^{N} \Psi_\alpha(\mathbf{x})\, Q_\alpha$

error minimization: $\text{GWS}^N = \int_{\Omega_e} \Psi_\alpha(\mathbf{x})\, \mathrm{L}(q^N) d\tau \equiv 0$



FE discretization: $\Omega \approx \Omega^h = \cup_e \Omega_e$

$q^N \equiv q^h = \cup_e \{N(\mathbf{x})\}^T \{Q\}_e$

FE GWS^h: $[\text{ Matrix }]\{Q\} = \{b\}$

error quantization: refined Ω^h solutions