TFD.1 Fluid Dynamics, the Physics of Turbulence

Eulerian conservation principles DM + D**P**

basic scales are length and velocity $\Rightarrow \mathrm{Re} = \mathrm{UL}/\upsilon$

$\mathrm{Re} < \delta$, DM + D**P** $\Rightarrow$ stable, time – independent solutions

$\mathrm{Re} > \delta$, DM + D**P** $\Rightarrow$ time dependent solutions characterized by

$$\left.\begin{array}{r}\text{cascades of vortices}\\ \text{cascade of length scales}\end{array}\right\}\text{"turbulence"}$$

Turbulent fluid motion

"*turbulent fluid motion is an irregular condition of flow in which the various measures show a random variation in time and space such that statistically distinct average values can be discerned*," (Hinze, 1975)

$\Rightarrow$ turbulence is a *continuum* phenomena
develops as an instability of laminar flow

TFD.2 Fluid Dynamics, the Physics of Turbulence

General properties of turbulence

- length scales range from domain size (L) to small (η)
- generates appearance of an "eddying" motion
- large scale (L) motion carries most of the energy
  responsible for enhanced diffusion processes
- smallest scale (η) motion >> molecular mean free path
  most vorticity resides in η scale motion
- turbulence energy cascades from L $\Rightarrow$ η scales
  turbulent flows are always dissipative
- turbulent flow state depends on upstream flow history
  local strain-rate tensor description not unique

TFD.3 Fluid Dynamics, the Physics of Turbulence

Kolmogorov scales in turbulent flow

all turbulent flows transfer turbulence kinetic energy k from L to smaller scales

conversion to heat via viscosity υ occurs only at the smallest scales

dissipation at the smaller scales depends only on

$\varepsilon = -\mathrm{d}k/\mathrm{d}t$, larger eddy energy supply rate

$\upsilon =$, fluid kinematic viscosity

Kolmogorov scales of turbulence

$$\text{for: } \varepsilon = D\left(\text{length}^2/\text{time}^3\right), \ \upsilon = D\left(\text{length}^2/\text{time}\right)$$

$$\text{length: } \eta = \left(\upsilon^3/\varepsilon\right)^{1/4}$$

$$\text{time: } \tau = \left(\upsilon^3/\varepsilon\right)^{1/2}$$

$$\text{velocity: } v = \left(\upsilon\varepsilon\right)^{1/4}$$

note: at STP, $\eta \approx O(10^2)$ times molecular mean free path

TFD.4 Fluid Dynamics, the Physics of Turbulence

The energy spectrum of turbulent flows

spectrum variable is wave number κ

$\lambda = 2\pi/\kappa$ is wave length

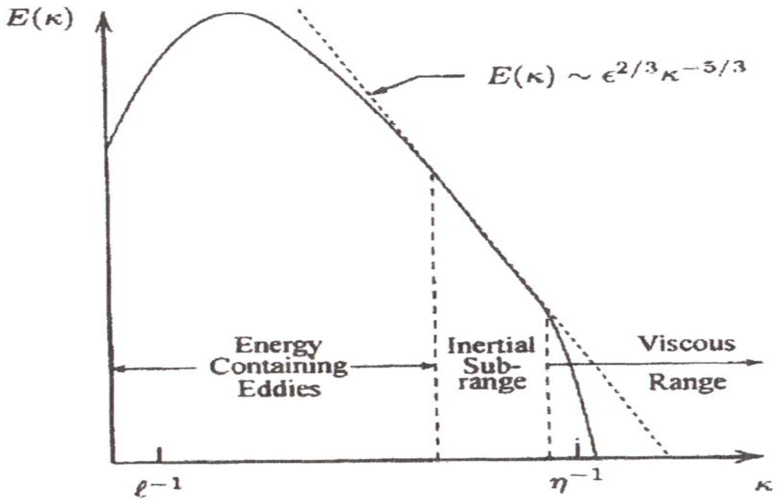
$\propto$ "eddy size"

turbulence kinetic energy

$$k \equiv \int_0^\infty E(\kappa)\mathrm{d}\kappa$$

$\mathrm{E}(\kappa)$ is energy spectral density

(related to Fourier transform)



Dimensional analysis leads to the following observations / correlations

$\varepsilon \sim k^{3/2}/\ell$ and $\ell >> \eta$

$$\frac{\ell}{\eta} = \frac{\ell}{\left(\upsilon^3/\varepsilon\right)^{1/4}} \sim \frac{\ell\left(k^{3/2}/\ell\right)^{1/4}}{\upsilon^{3/4}} \sim \mathrm{Re}_T^{3/4}\ ,\quad \mathrm{Re}_T \equiv \frac{k^{1/2}\ell}{\upsilon} >> 1$$

$$\therefore\ E(\kappa) \Rightarrow E(\varepsilon,\kappa) = \mathrm{C}_\kappa \varepsilon^{2/3}\kappa^{-5/3} \quad ,\ \frac{1}{\ell} << \kappa << \frac{1}{\eta}$$

TFD.5 Fluid Dynamics, the Physics of Turbulence

Perturbation theory removes arbitrariness of dimensional analysis

inertial region: $E(\kappa) \Rightarrow E(\varepsilon, \kappa, \upsilon) = \varepsilon^{1/4} \upsilon^{5/4} f(\kappa\eta)$

outer region: $E(\kappa) = k\ell g(\kappa\ell)$ in small wave number region

matching: $\varepsilon^{1/4} \upsilon^{5/4} f(\kappa\eta) = k\ell g(\kappa\ell)$, $\kappa\eta << 1$, $\kappa\ell >> 1$

"*between viscous and energetic turbulent flow scales an overlap region must exist wherein solutions match as* $\mathrm{Re} \Rightarrow \infty$"

enforcing smooth matching in wave number space, $\mathrm{d}(\cdot)/\mathrm{d}\,\kappa$ must be continuous

$$\Rightarrow \eta\varepsilon^{1/4} \upsilon^{5/4} f'(\kappa\eta) = \ell^2 k g'(\kappa\ell), \ \kappa\eta << 1, \ \kappa\ell >> 1$$

recalling: $\eta = \upsilon^{3/4} \varepsilon^{-1/4}$, $k = \varepsilon^{2/3} \ell^{2/3}$

$$\Rightarrow (\kappa\eta)^{8/3} f'(\kappa\eta) = (\kappa\ell)^{8/3} g'(\kappa\ell) = \text{constant}$$

since: $\kappa\eta$ and $\kappa\ell$ are independent variables

integrating

$$\Rightarrow f(\kappa\eta) = \mathrm{C}_\kappa \varepsilon^{2/3} \kappa^{-5/3}$$

TFD.6 Fluid Dynamics, Turbulent BL Similarity

Turbulent boundary layer near wall data exhibits self-similar log profile

$$\text{law-of-the-wall}: \ U/u_\tau = \kappa^{-1} \ln y^+ + C, \ \ y^+ = u_\tau y/\upsilon$$

Dimensional analysis provides insight

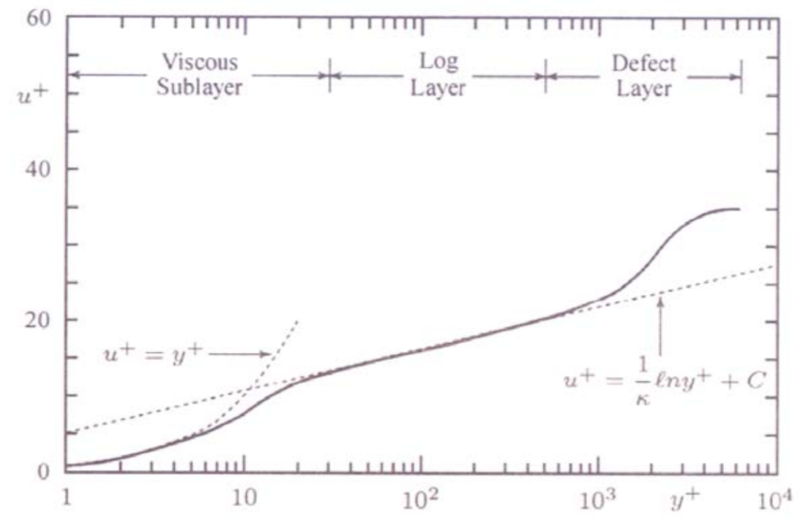
momentum exchange to wall $\Rightarrow \tau$

near wall $\tau(y) \approx$ constant $\Rightarrow \tau_w$

similar profiles for liquids and gases

$$\frac{\tau_w}{\rho} \equiv u_\tau^2 \Rightarrow D(\text{length}^2/\text{time}^2)$$

$$\frac{\mu}{\rho} = \upsilon \Rightarrow D(\text{length}^2/\text{time})$$



mean flow profile near wall, for velocity and length scales $u_\tau = \sqrt{\tau_w/\rho}$ and υ/u_τ

$$\frac{\partial U}{\partial y} = \frac{u_\tau}{y} F(u_\tau y/\upsilon) = \frac{u_\tau}{y} F(y^+)$$

data correlations confirm $F\,(u_\tau y/\upsilon) \Rightarrow \kappa^{-1}$ as υ influence vanishes, i.e., $y^+ \Rightarrow \infty$

$$\text{integrating}: \ \frac{U(y)}{u_\tau} = \frac{1}{\kappa} \ln\left(\frac{u_\tau y}{\upsilon}\right) + C, \ \ C \cong 5.0, \ \ \kappa \cong 0.41$$

TFD.7 Fluid Dynamics, Turbulent BL Characterization

Turbulent BL data confirm three distinct "flow physics" regions

$$\text{viscous sub-layer: } u^+ \approx y^+ \quad , \quad 1 \le y^+ < 5$$

$$\text{log layer: } u^+ = \kappa^{-1} \ln(y^+) + C \quad , \quad 30 \le y^+ \le 600$$

$$\text{defect layer: } u^+ \text{ not correlated by } \ln(y^+) \, , \quad y^+ > 600$$

Law-of-the-wall intercept depends on wall roughness

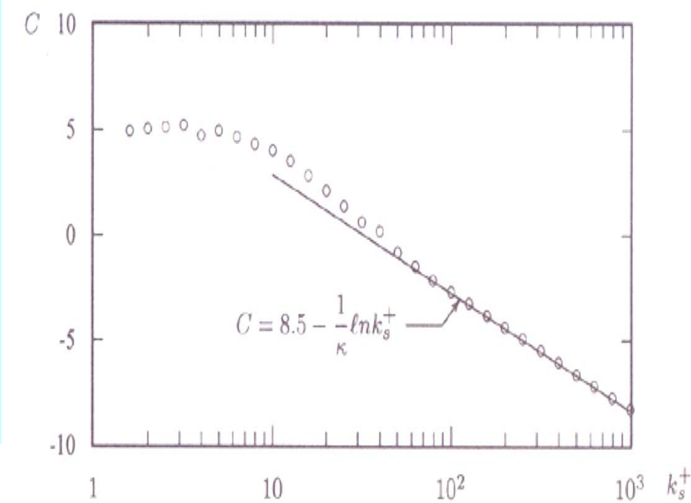
$$\text{non-D roughness height: } k_s^+ \equiv \frac{u_\tau k_s}{\upsilon}$$

$$\text{Nikuradse data: } C \to 8.5 - \kappa^{-1} \ln\left(k_s^+\right), \quad k_s^+ >> 1$$

$$\text{completely rough wall: } u^+ = \frac{1}{\kappa} \ln\left(\frac{y}{k_s}\right) + 8.5$$

note: is independent of viscosity υ

has impact when considering LES BCs



TFD.8 Fluid Dynamics, Turbulent BL Characterization

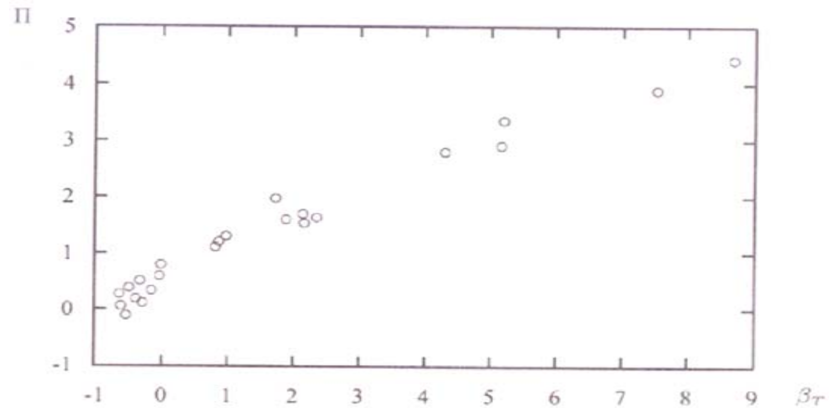
Boundary layer defect layer lies between log layer and BL edge (δ)

$$\text{data correlation}: \; u^+ = \frac{1}{\kappa}\ln(y^+) + C + \frac{2\Pi}{\kappa}\sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

Cole's wake parameter Π:

$$\beta_T \equiv \frac{\delta^* dP}{\tau_w dx}$$

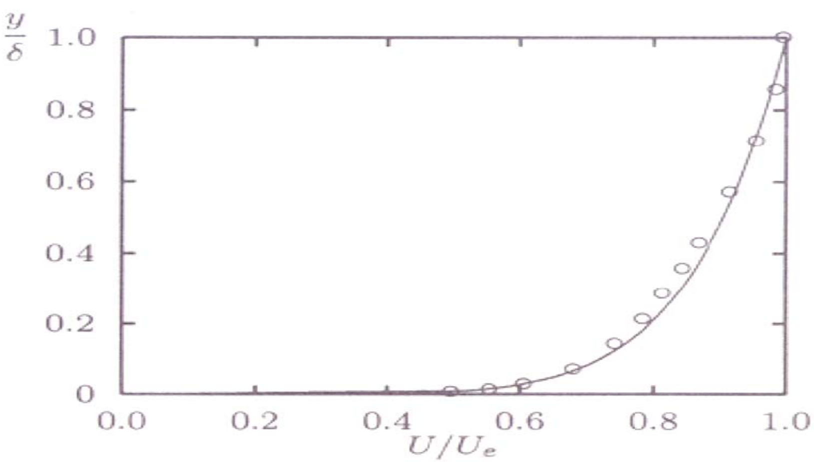
$\delta^* \equiv$ BL displacement thickness



Power law approximation

turbulent BL interpolation:

$$\frac{U}{U_e} \cong \left(\frac{y}{\delta}\right)^{1/n}, \; \text{for } n = 7 \rightarrow$$



TFD.9 Fluid Dynamics, Conservation Principle Manipulations

Velocity field resolved into mean and fluctuating components

resolution: $u_i(\mathbf{x},t) \equiv U_i(\mathbf{x},t) + u_i'(\mathbf{x},t)$

time – averaging for stationary turbulent flow

$$\bar{u}_i(\mathbf{x}) \equiv U_i(\mathbf{x}) \equiv \lim_{\mathrm{T}\to\infty} \frac{1}{\mathrm{T}} \int_t^{t+T} u_i(\mathbf{x},\tau)\,\mathrm{d}\tau$$

$$\bar{u}_i' \equiv \lim_{\mathrm{T}\to\infty} \frac{1}{\mathrm{T}} \int_t^{t+T} \left(u_i(\mathbf{x},\tau) - U_i(\mathbf{x},\tau)\right)\mathrm{d}\tau = 0$$

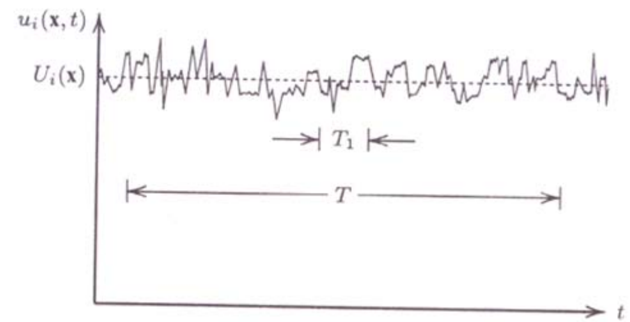
time – averaging for non – stationary turbulent flow

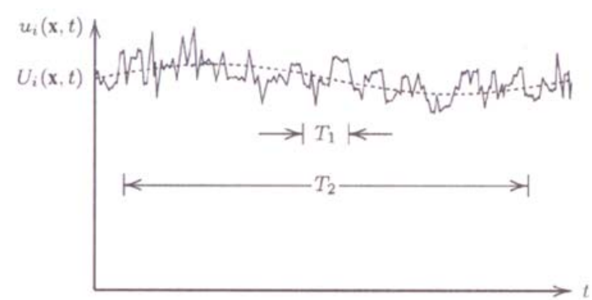
$$u_i(\mathbf{x},t) = \frac{1}{\mathrm{T}} \int_t^{t+T} u_i(\mathbf{x},\tau)\mathrm{d}\tau, \quad \mathrm{T}_1 \le \mathrm{T} \le \mathrm{T}_2$$

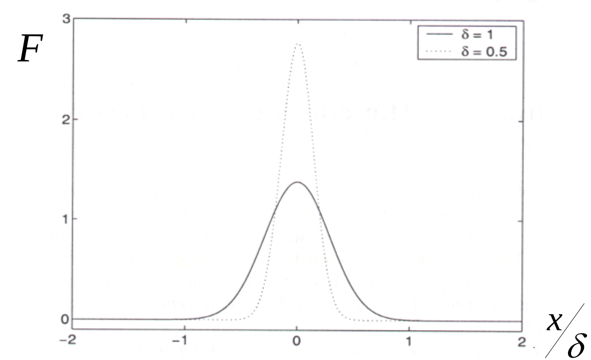
space – filtering for non – stationary turbulence

$$\bar{f}(\mathbf{x},t) = F * f(\mathbf{x},t) \equiv \int_\Omega F(\mathbf{x}-\mathbf{z})\, f(\mathbf{z},t)\mathrm{d}\mathbf{z}$$

$$\text{gaussian filter}: F_i(x_i) \equiv \sqrt{\frac{\gamma}{\pi\delta^2}}\exp\left(\frac{-\gamma}{\delta^2}x_i^2\right)$$







TFD.10 Fluid Dynamics, Conservation Principle Manipulations

Convection term in DP leads to Reynolds stress tensor

$$\text{time-averaging}: \left(\overline{\mathbf{uu}}\right)_t \Rightarrow \bar{\mathbf{u}}\bar{\mathbf{u}} + \overline{\mathbf{u}'\mathbf{u}'}$$

$$\text{space-filtering}: \left(\overline{\mathbf{uu}}\right)_\Omega \Rightarrow \overline{\bar{\mathbf{u}}\bar{\mathbf{u}}} + \overline{\bar{\mathbf{u}}\mathbf{u}'} + \overline{\mathbf{u}'\bar{\mathbf{u}}} + \overline{\mathbf{u}'\mathbf{u}'}$$

Closure requirement is to correlate fluctuating averages to state variable

$$\text{MLT algebraic}: \overline{u'v'} \cong \upsilon^t \frac{\partial U}{\partial y}\ , \ \upsilon^t \equiv (\omega\ell)^2 \left|\frac{\partial U}{\partial y}\right|$$

$$\text{PNS algebraic}: \overline{u_i'u'}_j \cong \frac{2}{3}k\delta_{ij} - \upsilon^t S_{ij}\ , \ S_{ij} = U_{i,j} + U_{j,i}$$

$$\text{TKE differential}: \mathsf{L}(k,\varepsilon) = 0 \ \text{added}, \ v^t = C_\mu k^2/\varepsilon$$

Turbulence intensities

flat plate BL

$$k = \frac{1}{2}\left[\overline{u'u'} + \overline{v'v'} + \overline{w'w'}\right]$$